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BRIEF COMMUNICATION

EXPLICIT EXPRESSIONS FOR VOLUME OF DROPS RELEASED FROM SUBMERGED NOZZLES: THEIR DERIVATION FROM SEMIEMPIRICAL IMPLICIT CORRELATIONS

Y. H. MORI and T. MOCHIZUKI
Department of Mechanical Engineering, Keio University, 3-14-1 Hiyoshi, Kohoku-ku, Yokohama 223, Japan

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INTRODUCTION

When a liquid flows out of a vertically-oriented cylindrical nozzle (or an orifice) into a medium of an immiscible liquid at a relatively low velocity, the former liquid disintegrates into uniform-sized drops, which separate one by one from the tip of the nozzle. There are several mechanistic models for such drop formation (Hayworth & Treybal 1950; Scheele & Meister 1968; Heertjes *et al.* 1971; de Chazal & Ryan 1971). The models individually give correlations for drop volume but in implicit forms only, so that iterative calculation procedures or graphical solutions have been considered necessary for solving the correlations to achieve actual evaluations of drop volume. In this communication we show that two well-known correlations can be transformed into explicit expressions for drop volume; the transformation is exact with Hayworth & Treybal's (1950) classical correlation but approximate with Scheele & Meister's (1968) correlation.

SPECIFICATION OF THE PROBLEM

When the flow rate of the drop-forming liquid is infinitesimal, the volume of each drop to separate from the nozzle tip, V_0, is given by

$$V_0 = \psi V_p, \qquad [1]$$

where V_p denotes the volume of a pendant drop on which the gravitational force resulting from the density difference $\Delta\rho$ between the two liquids and the interfacial tension σ multiplied by the circumference of the three-phase contact circle are balanced, i.e.

$$V_p = \frac{\pi D_N \sigma}{\Delta\rho g}, \qquad [2]$$

where D_N denotes the nozzle inside diameter which is assumed to agree with the contact circle diameter. The Harkins–Brown correction factor, ψ, is accurately expressed, in the range $D_N(\psi/V_0)^{1/3} \leqslant 1.4$, by the following correlation (Mori 1990):

$$\psi = 0.6 + 0.4\left[1 - \frac{D_N}{1.4}\left(\frac{\psi}{V_0}\right)^{1/3}\right]^{2.2}. \qquad [3]$$

Note that in [3]

$$\frac{\psi}{V_0} = \frac{\Delta\rho g}{\pi D_N \sigma}, \qquad [4]$$

and hence one can calculate V_0 with no trial-and-error, using the above set of equations.

As the flow rate of the drop-forming liquid increases, the volume of each drop released from the nozzle tip, V, deviates from V_0. Every correlation for V now available has, as mentioned in Introduction, an implicit form; or though it may appear to be explicit, it involves the Harkins–Brown correction factor which is not given by [3] this time but should be written as

$$\psi = 0.6 + 0.4\left[1 - \frac{D_N}{1.4}\left(\frac{\psi}{V}\right)^{1/3}\right]^{2\,2}. \qquad [5]$$

Hence it is generally appreciated that every correlation needs to be solved for V through an iterative calculation.

SOLUTION OF HAYWORTH & TREYBAL'S (1950) CORRELATION

The final form of the correlation that Hayworth & Treybal (1950) developed is as follows:

$$V + \upsilon V^{2/3} = \chi, \qquad [6]$$

where

$$\upsilon = 4.11 \times 10^{-4}\left(\frac{\rho_d U_N^2}{\Delta\rho}\right)$$

$$\chi = 2.1 \times 10^{-3}\left(\frac{\sigma D_N}{\Delta\rho}\right) + 1.069 \times 10^{-2}\left(\frac{D_N^{0.747} U_N^{0.365} \eta_c^{0.186}}{\Delta\rho}\right)^{3/2},$$

upon condition that we write the density of drop-forming liquid, ρ_d, and the density difference between the two liquids, $\Delta\rho$, in g/cm^3, the dynamic viscosity of the continuous-phase liquid, η_c, in cP (=mPa·s), σ in dyne/cm (=mN/m), the average flow velocity in the nozzle, U_N, in cm/s, and V in cm^3. Note that [6] results when the correlation is associated with an approximation that $\psi = 0.655$. It is suggested in Hayworth & Treybal's (1950) original article and also later publications referring to [6] (Treybal 1963; Steiner & Hartland 1983) that either an iterative calculation or a much simpler graphic means is necessary to solve [6] for V.

It appears quite curious that no one has pointed out, to our knowledge, that [6] has an analytic solution which is usable in practice. Only an elementary algebra is necessary to derive the solution. Equation [6] can readily be transformed into a simple cubic equation as

$$V^3 + (\upsilon^3 - 3\chi)V^2 + 3\chi^2 V - \chi^3 = 0. \qquad [7]$$

The application of the ordinary solution procedure for cubic equations to [7] yields

$$V = \Gamma_1 + \Gamma_2 + \chi - \tfrac{1}{3}\upsilon^3, \qquad [8]$$

where

$$\Gamma_1 = (\Lambda_1^{1/2} - \Lambda_2)^{1/3}, \qquad \Gamma_2 = -(\Lambda_1^{1/2} + \Lambda_2)^{1/3}, \qquad [9a]$$

$$\Lambda_1 = \tfrac{1}{4}\upsilon^6\chi^4 - \tfrac{1}{27}\upsilon^9\chi^3 = \left(\frac{\upsilon^3\chi^2}{2}\right)^2 - \left(\frac{\upsilon^3\chi}{3}\right)^3, \qquad [9b]$$

and

$$\Lambda_2 = \tfrac{1}{2}\upsilon^3\chi^2 - \tfrac{1}{3}\upsilon^6\chi + \tfrac{1}{27}\upsilon^9 = \upsilon^3\chi\left(\frac{\chi}{2} - \frac{\upsilon^3}{3}\right) - \left(\frac{\upsilon^3}{3}\right)^3, \qquad [9c]$$

upon condition that

$$\Lambda_1 > 0, \quad \text{i.e. } (\upsilon/3)^3 < \chi/4. \qquad [10]$$

This analytic solution is usable unless U_N exceeds a critical velocity at which Λ_1 changes its sign from positive to negative. The critical velocity can be a little lower than 0.3 m/s, the limit of applicability of [6] stated by Hayworth & Treybal (1950) themselves.

SOLUTION OF SCHEELE & MEISTER'S (1968) CORRELATION

Scheele & Meister's (1968) correlation is written as

$$\frac{V}{\psi}\Delta\rho g=\pi D_N\sigma+\frac{10.2\eta_c D_N^3 U_N}{V^{2/3}}-\frac{\pi}{3}\rho_d D_N^2 U_N^2+3.83D_N^2(\Delta\rho g\rho_d\sigma U_N^2)^{1/3}. \qquad [11]$$

There is no analytic means to solve, for V, the above correlation as it is, because: (i) V is involved both in the body force term (the l.h.s.) and the drag force term (the second term on the r.h.s.), carrying different exponents; and (ii) ψ is dependent on V as formulated by [5]. Thus, we need some tricks to deal with these two difficulties individually and thereby simplify the correlation.

We have empirically found that use of [3], combined with [4], instead of [5] in solving iteratively [11] rarely leads to more than a slight error in the V finally obtained. Thus, we employ this primitive trick—the replacement of the variable ψ with its asymptotic value for $U_N \to 0$—and thereby avoid one of the two difficulties in simplifying [11]. [The same trick was once used by Horvath *et al.* (1978) and then Steiner & Hartland (1983) but with a correlation for ψ different from [5].]

In formulating the drag force term finally into the form seen in [11], Scheele & Meister (1968) introduced various mechanistic approximations and evaluated empirically some numerical coefficients developed from such approximations. Inspecting the whole formulation process, we suppose that substituting a constant volume such as V_0 or V_p for V in the drag force term will not cause a relatively large loss of accuracy of the correlation, provided that the lead constant in the drag force term is adequately changed. For example, [11] may be rewritten as

$$\frac{V}{\psi}\Delta\rho g=\pi D_N\sigma+10.2c\frac{\eta_c D_N^3 U_N}{V_p^{2/3}}-\frac{\pi}{3}\rho_d D_N^2 U_N^2+3.83D_N^2(\Delta\rho g\rho_d\sigma U_N^2)^{1/3}, \qquad [12]$$

where V_p in the drag force term is given by [2] and the constant c needs to be determined empirically. Equation [12] is nondimensionalized as

$$\frac{V}{V_0}=\frac{V}{\psi V_p}=1+C\mathrm{Ca_N}\mathrm{Eo_N^{2/3}}-\tfrac{1}{3}\mathrm{We_N}+1.22(\mathrm{Eo_N}\mathrm{We_N})^{1/3}, \qquad [13]$$

where

$$C=10.2\,\pi^{-5/3}c, \qquad [14]$$

$$\mathrm{Ca_N}=\frac{\eta_c U_N}{\sigma} \quad \text{(nozzle capillary number)}, \qquad [15]$$

$$\mathrm{We_N}=\frac{\rho_d D_N U_N^2}{\sigma} \quad \text{(nozzle Weber number)}, \qquad [16]$$

$$\mathrm{Eo_N}=\frac{\Delta\rho g D_N^2}{\sigma} \quad \text{(nozzle Eötvös number)} \qquad [17]$$

and, as noted before, ψ can be calculated as

$$\psi=0.6+0.4\left[1-\frac{D_N}{1.4}\left(\frac{\Delta\rho g}{\pi D_N\sigma}\right)^{1/3}\right]^{22}. \qquad [18]$$

Once C is determined, [13] accompanied by [15]–[18] and [2] constitutes a closed-form solution for V. Based on some trial-and-error calculations, we recommend setting c at 0.735 and hence C at 1.11.

The validity of the approximate solution derived above is examined by comparing its predictions with the corresponding numerical solutions of [11]. The comparison is shown in table 1 for 3 drop/medium systems selected from the 15 systems listed with their physical properties data in Scheele & Meister's (1968) article. Heptane/water represents standard hydrocarbon/water systems. Heptane/glycerol is characterized by a quite high viscosity of the medium liquid. Butyl alcohol/water has an excessively low interfacial tension ($\lesssim 2$ mN/m). Three different nozzle diameters are assumed for each system. The U_N range for the comparison extends up to the jetting velocity

Table 1 The maximum fractional deviation (as a percentage) of the approximate solution for V from the corresponding exact numerical solution of Scheele & Meister's (1968) correlation

	D_N (mm)		
Drop/medium	0.3	1.5	3.0
Heptane/water	2.1	2.2	5.0
Heptane/glycerol	6.1	17.0	23.9
Butyl alcohol/water	1.3	7.2	—

predicted by Scheele & Meister's (1968) correlation for it. For the largest nozzle diameter, 3 mm, applied to the butyl alcohol/water system, the correlation does not predict a finite positive jetting velocity, and hence the comparison is not made here.

The agreement between the approximate and numerical solutions is reasonably good for the heptane/water and butyl alcohol/water systems, but relatively poor for the heptane/glycerol system. It should be stressed, however, that even the largest discrepancy between the approximate and numerical solutions shown cannot be called large compared with the uncertainty inherent to the original correlation [11] itself, the magnitude of which can be estimated roughly in view of some later reexaminations of the correlation (de Chazal & Ryan 1971; Steiner & Hartland 1983). In this context, the approximate solution proposed above is, we believe, of practical use.

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